DETERMINATION OF THE $\pi\pi$ SCATTERING PHASE SHIFTS UP TO 1.3 GeV C.M. ENERGY

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There is now considerable information available on various aspects of the elastic $\pi\pi$ interaction such as angular distributions, cross sections and S-wave contributions at low energies. Most of this information comes from the application of the one pion exchange model [1, 2] to the experimental results on single and double pion production in πp collisions such as $\pi^- p \to \pi^-\pi^0 p$ or $\pi^- p \to N^{*++}_{33}\pi^-\pi^-$ [3-13]. When the formfactor and off-shell correction functions of Ferrari and Selleri are used [14], the one pion exchange model yields consistent results on the $\pi\pi$ interaction from different reactions and with different momenta of the incoming pion [6-11, 13]. This gives confidence in those data on $\pi\pi$ scattering which have been obtained in this way.

In the study reported here we tried to combine all experimental data on the elestic $\pi\pi$ interaction and to analyse them in terms of the $\pi\pi$ scattering phase shifts. The main results of the analysis are: 1. A set of phase shifts is obtained which in a consistent way fit the existing data on $\pi\pi$ scattering. 2. The width of the ρ meson turned out to be 170 MeV, a value which has to be compared with the commonly accepted one of 110 MeV. 3. The existence of a $T = 0$, $J = 0$ $\pi\pi$ resonance is confirmed. A mass of 0.74 GeV and a full width at half maximum of 90 MeV were found for this resonance. A more detailed description of the analysis will appear elsewhere [15]. The values of the $\pi\pi$ phase shifts from threshold up to a c.m. energy of ω = 1.3 GeV are summarized in fig. 1.

With the help of the one pion exchange model, using the corrections proposed by Ferrari and Selleri, $\sigma_{\pi\pi}$, the total cross sections for elastic $\pi^-\pi^0$, $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ scattering have been determined from the reactions $\pi^- p \to \pi^-\pi^0 p$ (1.59 [6], 2.75 [7], 4.0 [10]), $\pi^- p \to \pi^+\pi^- n$ (1.59 [6], 2.75 [7], 4.0 [9-11, 16]), $\pi^+ p \to \pi^+\pi^+ n$ (4.0 [13]) and $\pi^- p \to N^{*++}_{33}\pi^-\pi^-$ (4.0 †). The figures in brackets give the laboratory momenta of the incident pion at which the reactions have been studied.

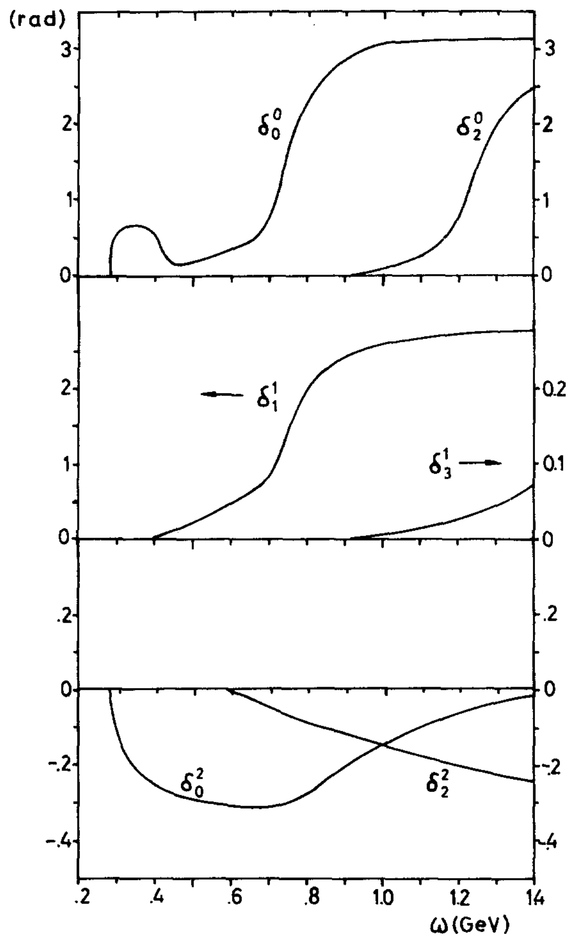

Fig. 1. $\pi\pi$ phase shifts δ^T_J as a function of the $\pi\pi$ c.m. energy ω.

† In ref. 8 $\sigma\pi^-\pi^-$ was determined without the use of off-shell corrections for the $\pi\pi$ vertex. The corresponding influence on the values of $\sigma_{\pi^-\pi^-}$ was estimated and the corrected values were used in this analysis.

Further information on $\pi\pi$ scattering comes from the measurements of a) the $\pi\pi$ angular distribution $W(\cos\theta)$, where θ is the angle between the incoming pion and the outgoing pion of the same charge in the $\pi\pi$ rest system; b) the asymmetry parameter R = (Forward - Backward)/(Forward + Backward). Apart from the above cited experiments there are data available from the reactions $\pi^- p \rightarrow \pi^-\pi^0 p$ (3.0 [17], 3.3 [18]), $\pi^- p \rightarrow \pi^+\pi^- n$ (3.0 [17], 3.3 [18], 6.0 [19]), $\pi^+ n \rightarrow \pi^+\pi^- p$ (6.0 [20]) and $\pi^- p \rightarrow N_{33}^{*++}\pi^-\pi^-$ (2.75 [8]).

According to Selleri [14] the $\pi\pi$ angular distribution is altered if one of the pions is off the mass shell and partial waves of different angular momentum states contribute. Therefore the $\pi\pi$ angular distribution measured in reactions such as $\pi N \rightarrow \pi\pi N$ may differ from the angular distribution for on-shell $\pi\pi$ scattering. The corrections for R are typically of the order of 10-30% for momentum transfers squared of $-t = 10\ \mu^2$ (μ pion mass). They have been taken into account in this analysis.

The $\pi\pi$ phase shifts δ^T_J were determined by first analysing $\pi^\pm\pi^\pm$ scattering which involves only the $T = 2$ isospin amplitude. The $\pi^\pm\pi^\pm$ cross section and the angular distribution could be well described assuming S and D waves only. The relative sign of δ^2_0 and δ^2_2 turned out to be positive and the negative value of $R_{\pi^-\pi^0}$, the asymmetry parameter for $\pi^-\pi^0$ scattering, determined the negative sign of δ^2_0.

Knowing the $T = 2$ phase shifts, the experimental results on $\pi^-\pi^0$ scattering were used to obtain the $T = 1$ phase shifts. The phase shift δ^1_1 is dominated by the ρ meson. The fit of the ρ width to the behaviour of $R_{\pi^-\pi^0}$ and of $\sigma_{\pi^-\pi^0}$ gave a full width at half maximum of 170 MeV. The phase shift δ^1_3 was found to be small.

The $T = 0$ phase shifts were then fitted using the $\pi^+\pi^-$ scattering data. At very low $\pi\pi$ masses the $T = 0$, S-wave $\pi\pi$ interaction was measured by studying the reaction $p + d \rightarrow {}^3He + 2\pi$ [21]. At large $\pi\pi$ masses ($\omega \gtrsim 1$ GeV) there is a strong D-wave contribution which mainly comes from the f meson. We therefore calculated δ^0_2 using a resonant phase-shift formula [22]. Knowing δ^0_2 one can now try to determine δ^0_0 for $\omega > 0.5$ GeV. Whereas in this region $\sigma_{\pi^+\pi^-}$ is insensitive to the value of δ^0_0, $R_{\pi^+\pi^-}$ depends critically on δ^0_0. Fitting δ^0_0 to $R_{\pi^+\pi^-}$ we found that the large asymmetry ($R_{\pi^+\pi^-} \approx 0.4 - 0.6$), observed from 0.6 GeV up to 0.9 GeV, can only be explained by assuming the existence of a resonance in the $T = 0$, $J = 0$ state with a mass of about that of the ρ meson. Best agreement was found with a mass of 0.74 GeV and a width of 90 MeV. This result has been suggested by several authors and has been found in similar investigations [23, 24].

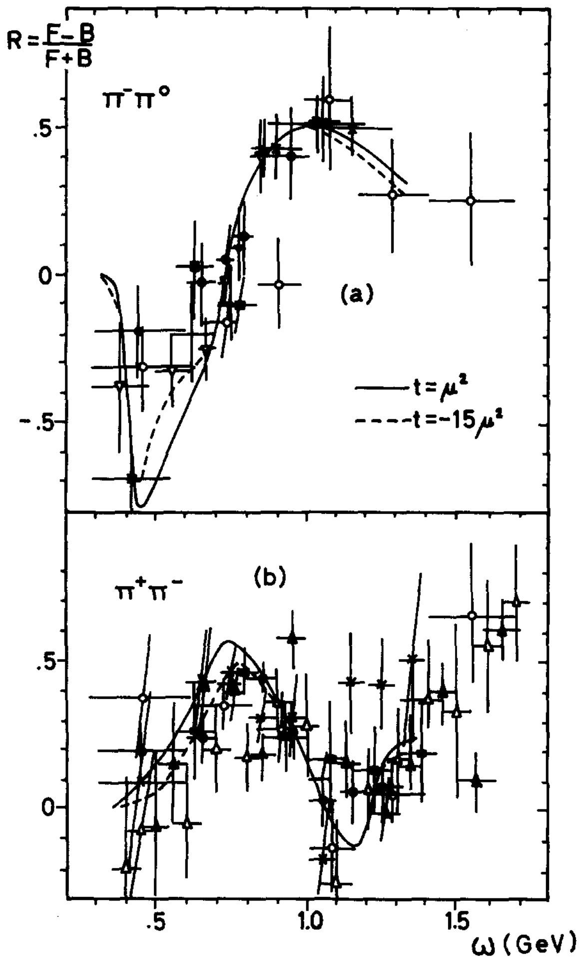

Fig. 2. Energy dependence of the asymmetry parameter R. The data were taken from the compilation given in ref. 7 and from ref. 20. The curves shown were calculated from the δ^T_J for on-shell $\pi\pi$ scattering (solid line) and for the scattering of a real pion on a pion with a mass squared of $-15\ \mu^2$ (dashed line). a) $\pi^-\pi^0$ system, b) $\pi^+\pi^-$ system.

Fig. 1 gives the behaviour of the phase shifts. In figs. 2a, b, $R_{\pi^-\pi^0}$ and $R_{\pi^+\pi^-}$, calculated from the phase shifts are compared with the measurements. The values of R were calculated a) for on-shell $\pi\pi$ scattering, b) for momentum transfers squared to the $\pi\pi$ system $-t = 15\ \mu^2$ [25]. In figs. 3a-c, a comparison is made between the

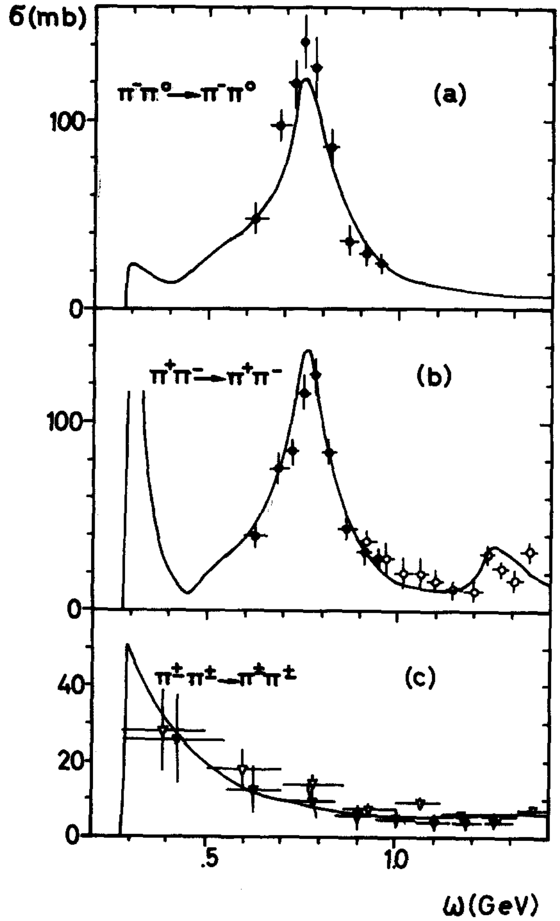

Fig. 3. Elastic $\pi\pi$ scattering cross section as a function of the $\pi\pi$ c.m. energy ω. a) $\pi^-\pi^0$ scattering. The data come from $\pi^-p \to \pi^-\pi^0p$ (1.59, 2.75 [7]). b) $\pi^+\pi^-$ scattering. The data come from $\pi^-p \to \pi^+\pi^-n$ (● 1.59 and 2.75 [7], o 4.0 [16]). c) $\pi^\pm\pi^\pm$ scattering. The data come from $\pi^+p \to \pi^+\pi^+n$ (∇4.0 [13]) and from $\pi p \to N_{33}^{*++}\,\pi^-\pi^-$ (▼4.0 †).

measured and the calculated $\pi\pi$ cross sections. The figures show that the experimental results are well described by the fitted values of the phase shifts.

† See footnote on page 328.

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